APRIL/MAY 2024

23PEMA14B — GRAPH THEORY AND APPLICATIONS

Shanmuga Industries Arts & Science College ★ T.V.Malai

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define isomorphism of graphs. Give an example.
2. Define induced subgraph of G.
3. Prove that $K'(G)=1$ if and only if G is connect graph with a cut edge.
4. Define closure of G. Give an example.
5. Define covering number of G. Give an example.
6. Define edge chromatic number. Give an example.
7. Prove that $\alpha+\beta=\nu$.
8. Define chromatic polynomial of G.
9. If G is a plane graph, show that $\sum_{f\in F} d(f)=2\epsilon$.
10. If G is a simple planar graph, prove that $8\geq 5$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Prove that every tree has at least two vertices of degree one.

Or

(b) Show that a vertex σ of a tree is a cut vertex of G if and only if $d(\sigma) > 1$.

12. (a) Prove that $K(G) \leq K'(G)$.

Or

(b) Show that a connected graph G has an euler trail if and only if it has at most two vertices of odd degree.

13. (a) Prove that every 3-regular graph without cut edges has a perfect matching.

Or

(b) Show that for a bipartite graph G, $X'(G) = \Delta$.

14. (a) Prove that $r(k, l) \leq \binom{k+l-2}{k-l}$.

Or

(b) If G is K-critical show that $\delta(G) \geq K - 1$.

15. (a) Prove that K_5 is nonplanar.

Or

(b) If G is simple planar graph with $v \geq 3$, show that $\epsilon \leq 3v - 6$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that a graph G is bipartite if and only if G has no odd cycles.

17. If G is simple with $v(G) \geq 3$ and $\delta(G) \geq v/2$, prove that G is Hamiltonian.

18. State and prove Berge theorem for maximum matching.

19. State and prove Brook's theorem.

20. State and prove Euler formula for a plane graph.

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